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On Extending Hotelling's q-test for Complete Independence Between Two Sets of Variates

Tito A. Mijares[1]

ABSTRACT

The distribution of the product of sample canonical correlations $q=r_1r_2$ between two sets of variates when all population canonical correlations are zero had been found by Hotelling(1936) to have an exact beta disribution.This paper considers the disributions of the extended cases of complete independence when there are $s = 3$ & 4 variates in the first set and $t \geq 3$ & 4 variates in the second set by fitting moments of Pearson curves to the first four moments of variate products $(\theta_1\theta_2\theta_3)$ and $(\theta_1\theta_2\theta_3\theta_4)$,where $\theta_i = r_i^2$, the square of the i-*th* sample canonical correlation. Upper percent points of the s th roots of the s- variate products are presented for $s = 2,3$ and 4..

Keywords and Phrases: Hotelling's q-test, product of roots test, *sth* elementary symmetric function of s roots test, tests of complete independence between two sets of variates.

1. Introduction

Relations between two variables are ordinarily measured by *simple* correlations between them. In associations between a variable and a set of other variables two types of correlations are known: *multiple* and *partial* correlations. The latter is used to describe relations when one or more variables in the set assume fixed values (i.e., the conditional type).

In associations between two sets of variables we deal not only of relations between variables within the same set but also of relations of a variable in one set with each variable of the other set. The degree(s) of true relationship between a linear function of the first set and a linear function of the second set of variables, after effects of all correlations within each set have been removed, is (are) measured by their *canonical* correlation(s). Calculations of canonical correlations are discussed in many multivariate statistics textbooks (e.g., Morrison,1976; Kendall, 1980).

In this paper the number of variables in the first set is denoted by s while that of the second set the number is denoted by t , where s is less than or equal to t . It is assumed that the vector of$(s+t)$ variables has a population covariance matrix given by Σ,which is assumed to be positive definite and conformally partitioned into Σ_{11}, the covariance matrix of the first set of variables; $\Sigma_{12} = \Sigma_{21}$, the covariance matrix between the first and the second sets of variables, and Σ_{22}, the covariance matrix of the second set of variables. From Σ we obtain the population correlation matrix $\mathfrak{R}$ of the vector of $(s+t)$ variables.Thus, from a sample of size N greater than $(p+q)$ we can have an estimate of Σ, which we shall call *sample covariance matrix* **S** or an estimate of $\mathfrak{R}$, which we shall call *sample correlation matrix* **R**, both of which are conformally partitioned as their

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[1] National Academy of Science and Technology, Philippines

population counterparts. We shall refer to $0 \leq r_1 \leq r_2 \leq \ldots \leq r_s \leq 1$ as the *sample canonical correlations*, or canonical correlations calculated from the sample as estimates of the population canonical correlations ρ_i, $i=1,\ldots,s$, which are not necessarily indexed in the same order.

Testing the null hypthesis of complete independence between two sets of variates is equivalent to testing that all population canonical correlations are zero. In the case when $s=t=2$ Hotelling (1936) first proposed to test this hypothesis using what he called the vector sample correlation coefficient q, which is the product of the two sample canonical correlations; that is, $q= r_1 r_2$. Under null hypothesis he showed that the exact sampling distribution of q is an exact beta given in the form

$$\frac{(N-1)!}{(t-2)!(N-t)!} q^{t-2}(1-q)^{N-t-1}\,dq, \tag{1.1}$$

where N is the number of samples. As a generalization of Hotelling's result, Hsu (1939) obtained the joint density of the latent roots of certain determinantal equation in multivariate beta form given by

$$\mathrm{K} \bullet \left\{\prod_{i=1}^{s} \theta_i\right\}^{(t-s-1)/2} \left\{\prod_{i=1}^{s} (1-\theta_i)\right\}^{(N-s-t-2)/2} \prod_{i<j}^{s} (\theta_i - \theta_j) \tag{1.2}$$

where $t \geq s$, N is the total sample size, the roots $\theta_i = r_i^2$ and arranged in descending order of magnitude, and K is a constant such that the integral of (1.2) equals unity.

In this paper we will consider the cases of $s = 3$ & 4 and, in particular, give a set of tables for the upper percent points of the *s-th* root of the variate products $\theta_1\theta_2\ldots\theta_s$ for $s = 2,3$ & 4.

2. The Moments of the Product of s Roots

For our purpose we follow notations of parameters, symbols and order of magnitude of the roots and the constant K of those used in Pillai & Mijares (1959) and Mijares (1964). Thus, the joint density of (1.2) is given as

$$f(\theta_1,\ldots,\theta_s) = C(s,m,n)\prod_{i=1}^{s} \theta_i^{m}(1-\theta_i)^{n}\prod_{i>j}(\theta_i-\theta_j),\quad 0<\theta_1 \leq \ldots \leq \theta_s < 1, \tag{2.1}$$

where

$$C(s,m,n) = \frac{\pi^{s/2}\prod_{i=1}^{s}\Gamma\left[\left(\frac{1}{2}\right)(2m+2n+s+i+2)\right]}{\prod_{i=1}^{s}\Gamma\left[\left(\frac{1}{2}\right)(2m+i+1)\right]\Gamma\left[\left(\frac{1}{2}\right)(2n+i+1)\right]\Gamma\left(\frac{i}{2}\right)}.$$

The parameters m and n in (2.1) are equivalent to $(t - s - 1)/2$ and $(N - s - t - 2)/2$, respectively, in (1.2). The s in both refers to the number of non-zero roots.

Denote by $V_j^{(s)}$ the jth elementary symmetric function (e.s.f.) of the θ's. Mijares (1961) derived the general form of the moments and joint moments of the e.s.f.'s. For the case of the s th e.s.f. $V_s^{(s)}$ it can be checked easily from (2.1) that the αth moment of $V_s^{(s)}$ is

$$\mu_\alpha'\left(V_s^{(s)}\right)=\frac{C(s,m,n)}{C(s,m+\alpha,n)} \tag{2.2}$$

Table 1 below gives the first four raw moments of $V_s^{(s)}$ for $s=2,3$ and 4. The notation M(a,b,...) denotes the product $(2m$ + a)$(2m$ + b)...and P(a,b,...) denotes the product $(2p$ + a)$(2p$ + b)..., where $p = m + n$.

Table 1. First four raw moments of the sth e.s.f. of s roots, $V_s^{(s)}$, for s = 2, 3 & 4.

	$V_2^{(2)}$	$V_3^{(3)}$	$V_4^{(4)}$
μ_1'	M(2,3)/	M(2,3,4)/	M(2,3,4,5)/
	P(5,6)	P(6,7,8)	P(7,8,9,10)
μ_2'	M(2,3,4,5)/	M(2,3,4^2,5,6)/	M(2,3,4^2,5^2,6,7)/
	P(5,6,7,8)	P(6,7,8^2,9,10)	P(7,8,9^2,10^2,11,12)
μ_3'	M(2,3,4,5,6,7)/	M(2,3,4^2,5,6^2,7,8)/	M(2,3,4^2,5^2,6^2,7^2,8,9)/
	P(5,6,7,8,9,10)	P(6,7,8^2, 9,10^2,1112)	P(7,8,9^2,10^2,11^2,12^2,13,14)
μ_4'	M(2,3,4,5,6,7,8,9)/	M(2,3.4^2,5,6^2,7,8^2,9,10)/	M(2,3,4^2,5^2,6^2,7^2,8^2,9^2,10,11)/
	P(5,6,7,8,9,10,11,12)	P(6,7,8^2,9,10^2,11,12^2,13,14)	P(7,8,9^2,10^2,11^2,12^2,13^2,14^2,15,16)

From the table, the moment $\mu_j'(V_2^{(2)})$ may be observed to be equal to 2jth moment of variate $\sqrt{V_2^{(2)}}$; that is, $\mu_j'(V_2^{(2)}) = \mu_{2j}'(\sqrt{V_2^{(2)}})$. Thus, $\sqrt{V_2^{(2)}}$, which is equivalent to Hotelling's variate q, has an exact beta distribution with parameters $2m+2$ and $2n+3$. However, in the extended case of $s=3$ & 4, $\mu_j'(V_s^{(s)}) \neq \mu_{2j}'[(V_s^{(s)})^{1/s}]$. In the next section we investigate the distributions by fitting the moments of an approximate beta variate $(V_s^{(s)})^{1/s}$ and compare the moments of $V_s^{(s)}$ with those of the true distribution through moment ratios.

3. Moment Ratios of the Approximate Distributions of $V_s^{(s)}$, $s = 3$ & 4.

Since the expected values (Mijares, 1998)

$$E\{[V_s^{(s)}]^{1/s}\}^{\alpha s} = \mu_\alpha'(V_s^{(s)}), \tag{3.1}$$

the αth moment of $V_s^{(s)}$ can be approximated by moments of a fitted beta distribution to variate $\{V_s^{(s)}\}^{1/s}$. More specifically, consider $s=3$. Let "a" and "p" be parameters of the approximate beta distribution to be estimated. From Table 1, we have

$$\frac{M(2,3,4)}{P(6,7,8)} = \mu_1'(V_3^{(3)}) \tag{3.2}$$

$$\frac{M(4,5,6)}{P(8,9,10)} = \frac{\mu_2'\left(V_3^{(3)}\right)}{\mu_1'\left(V_3^{(3)}\right)} \tag{3.3}$$

The first approximation to parameters "a" and "p" may be computed by using (3.2) and (3.3). Thus,

$$\frac{a'+1}{p'+1} = \left[\mu_1'\left(V_3^{(3)}\right)\right]^{1/3} \tag{3.4}$$

$$\frac{a'+4}{p'+4} = \left[\frac{\mu_2'\left(V_3^{(3)}\right)}{\mu_1'\left(V_3^{(3)}\right)}\right]^{1/3} \tag{3.5}$$

We solve for a′ and p′ in terms of M's and P's. The second approximation consists of adjusting a′ by Δ such that $a = a' + \Delta$ and holding $p'=p$ without adjustment. Using only the first two terms of Taylor's expansion let $f_1(a) = a(a+1)(a+2)/p(p+1)(p+2)$, then

$$f_1(a) \approx f_1(a') + \Delta f_1'(a'), \tag{3.6}$$

where $f_1'(a')$,is the first derivative of $f_1(a)$ with respect to a at a′. From (3.2), (3.4) and (3.5), we have

$$\varepsilon = \left[\frac{\mu_1'\left(V_3^{(3)}\right)}{f_1(a')} - 1\right] \tag{3.7}$$

giving

$$\Delta = \varepsilon \times \frac{f_1(a')}{f_1'(a')}. \tag{3.8}$$

Values of approximate moment ratios $\beta_1 = \{[\mu_3(V_3^{(3)})]^2 / [\mu_2(V_3^{(3)})]^3$ and $\beta_2 = [\mu_4(V_3^{(3)})] / [\mu_2(V_3^{(3)})]^2$ calculated are compared with the moment ratios of the true distribution for selected values of the parameters *m* and *n* (Table 2). Similar calculations are made for the approximate moment ratios of $V_4^{(4)}$. The approximate moment ratios are compared with their true moment ratios (Table 3). From these tables the moment ratios obtained from the beta approximation may be observed to be very close (2-3 significant digit accuracy) to those of the true ones for a wide range of values of the parameters *m* and *n*.

Table 2. Values of the moment ratios $\beta_1 = \{[\mu_3(V_3^{(3)})]^2 / [\mu_2(V_3^{(3)})]^3$ and

$\beta_2 = [\mu_4(V_3^{(3)})] / [\mu_2(V_3^{(3)})]^2$ obtained from the beta approximation to the distribution of $V_3^{(3)}$ and those of the true ones in (*).

	2m = 5	*2m = 10*	*2m = 50*
2n = 10	β_1 = 2.52 (2.50) β_2 = 6.83 (6.81)	β_1 = 0.93 (0.92) β_2 = 4.25 (4.25)	β_1 = $0.0^4 4$ ($0.0^4 4$) β_2 = 2.87 (2.87)
2n = 50	β_1 = 5.86 (5.84) β_2 = 13.41 (13.35)	β_1 = 2.68 (2.68) β_2 = 7.57 (7.76)	β_1 = 0.23 (0.23) β_2 = 3.31 (3.31)

Table 3. Values of the moment ratios $\beta_1 = \{[\mu_3(V_4^{(4)})]^2 / [\mu_2(V_4^{(4)})]^3$ and $\beta_2 = [\mu_4(V_4^{(4)})] / [\mu_2(V_4^{(4)})]^2$ obtained from the beta approximation to the distribution of $V_4^{(4)}$ and those of the true ones in (*).

	2m = 5	*2m = 10*	*2m = 50*
2n = 10	β_1 = 4.27 (4.33) β_2 = 10.07 (10.17)	β_1 = 1.67 (1.67) β_2= 5.58 (5.57)	β_1 = 0.028 (0.028) β_2 = 2.93 (2.93)
2n = 50	β_1 = 9.66 (9.81) β_2 = 21.34 (21.64)	β_1 = 4.29 (4.29) β_2 = 10.68 (10.68)	β_1 = 0.39 (0.39) β_2 = 3.60 (3.60)

4. Upper Percent Points of *s-th* Roots of $V_s^{(S)}$.

From the approximations of Section 3 moment ratios of $\{V_s^{(s)}\}^{1/s}$ are computed for $s = 3$ & 4. Upper percent points are calculated for $m \geq 0$; i.e., $t \geq s-2$, by fitting Pearson curves (Table 42. Percentage points of Pearson curves for β_1, β_2 expressed in standardized measure; *Biometrika Tables for Statisticians*, Vol. 1, E.S.Pearson and H.O. Hartley, ed. 1956). These are shown in Tables A-2 & A-3, respectively. For completness, the percent points of q (i.e., square root of $V_2^{(2)}$) are given also in Table A-1 of the Appendix.

(**A note on the parameters.** Let s be the number of variates in the first set and t be the number of variates in the second set, where s is equal or less than t. Let N be the total size of the $(s+t)$ -normal sample. The parameters of the distribution of the canonical sample correlations are defined by

$$2m = t - s - 1$$
$$2n = N - t - s - 2.$$

Canonical sample correlations can be calculated from products of conformally partitioned matrices of either the sums of product and cross-product (S.P.) matrix **C**, the sample covariance matrix **S**, or the correlation matrix **R**. Thus, the squares of sample canonical correlations can be found from the eigenvalues of the product of conformally partitioned matrices $\mathbf{C}_{11}^{-1}\mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}$, $\mathbf{S}_{11}^{-1}\mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{21}$, or $\mathbf{R}_{11}^{-1}\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{R}_{21}$.)

As an illustration, Pillai (1957) has reported a study carried out for testing the independence of a set of three variates (1, systolic pressure; 2, diastolic pressure; 3,pulse beat) to another set of four variates (4, height; 5, weight; 6, chest, and 7, waist measurements) to a group of 60 male reserve officers belonging to the age group 29-31 of the Armed Forces of the Philippines. From the S.P. matrix of the first set of variates $\mathbf{C}_{11}$, the S.P. matrix of the second set $\mathbf{C}_{22}$, and the S.P. matrix of the first and second sets of variates $\mathbf{C}_{12}$ the product of the conformally partitioned matrices $\mathbf{C}_{11}^{-1}\mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}$, where $\mathbf{C}_{21}$ is the transpose of $\mathbf{C}_{12}$, is shown as follows

$$\begin{pmatrix} 0.10372357 & -0.06139690 & -0.02083534 \\ 0.02095806 & 0.02171374 & -0.00243910 \\ -0.08262453 & -0.06151884 & 0.05018319 \end{pmatrix}$$

The eigenvalues of this matrix turns out to be $\theta_1 = 0.00886503$, $\theta_2 = 0.0281732$ and $\theta_3 = 0.13858227$. To test the independence of the two sets of variates, Pillai used the largest root test which led him to accept the H_o since $\theta_3 < x_{.95}^{(3)}$, the table value. If we use the extended Hotelling q-test, the parameters are $2m = 0$ and $2n = 51$. The product $V_3^{(3)} = 0.0000346$, which is less than the approximate 5%-significance level table value of $(.081)^3 = 0.0005314$ in Table A-2. Hence,we accept the H_o of complete independence between the first set of three variates and the second set of four variates, which is consistent with the conclusion made by Pillai's largest root test.

5. Appendix

Table A-1. Upper Percentage Points of the Square Root of $V_2^{(2)}$

	2n=5	*2n=10*	*2n=15*	*2n=20*	*2n=24*	*2n=30*	*2n=40*	*2n=60*	*2n=120*
2m=-1 5%	0.399	0.264	0.196	0.156	0.134	0.111	0.086	0.059	0.030
1%	0.584	0.411	0.315	0.255	0.221	0.184	0.144	0.100	0.052
2m=0 5%	0.527	0.369	0.283	0.229	0.199	0.166	0.130	0.090	0.047
1%	0.683	0.507	0.400	0.330	0.289	0.243	0.192	0.136	0.072
2m=1 5%	0.603	0.440	0.345	0.283	0.247	0.208	0.164	0.115	0.061
1%	0.740	0.569	0.459	0.383	0.338	0.287	0.229	0.163	0.087
2m=2 5%	0.657	0.494	0.394	0.327	0.287	0.243	0.194	0.137	0.073
1%	0.777	0.615	0.504	0.425	0.378	0.323	0.260	0.187	0.101
2m=3 5%	0.697	0.537	0.435	0.364	0.322	0.274	0.220	0.157	0.085
1%	0.805	0.651	0.541	0.461	0.412	0.354	0.287	0.208	0.114
2m=4 5%	0.728	0.573	0.470	0.397	0.353	0.302	0.244	0.176	0.095
1%	0.826	0.680	0.572	0.491	0.441	0.382	0.312	0.228	0.125
2m=5 5%	0.753	0.604	0.500	0.426	0.380	0.328	0.266	0.193	0.106
1%	0.843	0.705	0.599	0.518	0.467	0.407	0.334	0.246	0.136
2m=6 5%	0.774	0.630	0.527	0.452	0.405	0.351	0.286	0.209	0.115
1%	0.857	0.725	0.622	0.542	0.491	0.429	0.354	0.262	0.147
2m=7 5%	0.792	0.652	0.551	0.475	0.428	0.372	0.306	0.225	0.125
1%	0.869	0.743	0.642	0.563	0.512	0.450	0.373	0.278	0.157
2m=8 5%	0.807	0.672	0.572	0.497	0.449	0.392	0.323	0.239	0.134
1%	0.879	0.759	0.660	0.582	0.531	0.468	0.391	0.293	0.167
2m=9 5%	0.820	0.690	0.592	0.516	0.468	0.411	0.340	0.253	0.143
1%	0.887	0.773	0.677	0.600	0.549	0.486	0.407	0.307	0.176
2m=10 5%	0.831	0.706	0.609	0.534	0.486	0.428	0.356	0.266	0.151
1%	0.894	0.785	0.692	0.616	0.565	0.502	0.423	0.320	0.185

Table A-2. Upper Percentage Points of the Cube Root of $V_3^{(3)}$

		2n=10	2n=20	2n=30	2n=40	2n=50	2n=60	2n=80	2n=100	2n=120
2m=0	5%	0.268	0.170	0.124	0.098	0.081	0.069	0.053	0.043	0.036
	1%	0.339	0.218	0.161	0.127	0.105	0.090	0.070	0.057	0.048
2m=1	5%	0.327	0.213	0.157	0.125	0.104	0.088	0.068	0.056	0.047
	1%	0.396	0.262	0.196	0.155	0.129	0.110	0.086	0.070	0.060
2m=2	5%	0.376	0.250	0.187	0.149	0.124	0.107	0.083	0.068	0.057
	1%	0.443	0.300	0.227	0.182	0.152	0.130	0.102	0.083	0.070
2m=3	5%	0.418	0.283	0.214	0.173	0.144	0.124	0.096	0.079	0.067
	1%	0.483	0.333	0.254	0.205	0.172	0.148	0.116	0.095	0.081
2m=4	5%	0.454	0.313	0.239	0.193	0.162	0.139	0.109	0.090	0.076
	1%	0.517	0.362	0.279	0.227	0.191	0.165	0.129	0.107	0.091
2m=5	5%	0.485	0.340	0.262	0.213	0.179	0.154	0.121	0.100	0.085
	1%	0.546	0.390	0.303	0.247	0.209	0.181	0.142	0.117	0.100
2m=6	5%	0.512	0.365	0.283	0.231	0.195	0.169	0.133	0.110	0.093
	1%	0.571	0.415	0.324	0.266	0.225	0.196	0.155	0.128	0.109
2m=7	5%	0.537	0.388	0.303	0.249	0.211	0.183	0.144	0.119	0.102
	1%	0.594	0.437	0.344	0.284	0.241	0.210	0.166	0.138	0.118
2m=8	5%	0.560	0.409	0.322	0.265	0.225	0.196	0.155	0.129	0.110
	1%	0.615	0.458	0.363	0.301	0.256	0.224	0.178	0.148	0.126
2m=9	5%	0.579	0.429	0.340	0.281	0.240	0.209	0.166	0.138	0.118
	1%	0.632	0.477	0.381	0.317	0.271	0.237	0.189	0.157	0.134
2m=10	5%	0.598	0.447	0.356	0.296	0.253	0.221	0.176	0.147	0.125
	1%	0.648	0.494	0.397	0.332	0.285	0.249	0.199	0.166	0.143
2m=12	5%	0.631	0.480	0.387	0.324	0.279	0.244	0.196	0.164	0.140
	1%	0.679	0.525	0.427	0.360	0.310	0.273	0.220	0.184	0.158
2m=14	5%	0.659	0.510	0.415	0.350	0.302	0.266	0.215	0.180	0.155
	1%	0.707	0.553	0.457	0.385	0.334	0.295	0.239	0.200	0.173
2m=16	5%	0.684	0.535	0.440	0.373	0.324	0.286	0.232	0.195	0.168
	1%	0.732	0.577	0.479	0.409	0.354	0.315	0.257	0.216	0.187
2m=18	5%	0.706	0.559	0.464	0.396	0.345	0.306	0.249	0.210	0.181
	1%	0.798	0.599	0.502	0.430	0.377	0.335	0.274	0.231	0.200
2m=20	5%	0.724	0.580	0.485	0.416	0.364	0.324	0.265	0.224	0.194
	1%	0.815	0.619	0.521	0.450	0.396	0.353	0.290	0.246	0.213

Table A-3. Upper Percentage Points of the Fourth Root of $V_4^{(4)}$

		2n=10	*2n=20*	*2n=30*	*2n=40*	*2n=50*	*2n=60*	*2n=80*	*2n=100*	*2n=120*
2m=0	*5%*	0.251	0.163	0.121	0.096	0.080	0.068	0.053	0.043	0.036
	1%	0.308	0.203	0.151	0.120	0.100	0.086	0.066	0.054	0.046
2m=1	*5%*	0.307	0.204	0.153	0.122	0.102	0.087	0.068	0.055	0.047
	1%	0.363	0.245	0.184	0.148	0.124	0.106	0.083	0.068	0.057
2m=2	*5%*	0.353	0.239	0.181	0.145	0.121	0.104	0.081	0.067	0.056
	1%	0.408	0.280	0.213	0.172	0.144	0.124	0.097	0.080	0.068
2m=3	*5%*	0.392	0.270	0.206	0.167	0.140	0.121	0.094	0.077	0.066
	1%	0.446	0.312	0.240	0.194	0.164	0.141	0.111	0.091	0.077
2m=4	*5%*	0.426	0.299	0.230	0.187	0.157	0.136	0.107	0.088	0.075
	1%	0.479	0.340	0.264	0.215	0.182	0.157	0.124	0.102	0.087
2m=5	*5%*	0.456	0.325	0.252	0.206	0.174	0.150	0.118	0.097	0.083
	1%	0.508	0.366	0.286	0.234	0.199	0.172	0.136	0.114	0.096
2m=6	*5%*	0.483	0.349	0.272	0.223	0.189	0.164	0.131	0.108	0.092
	1%	0.533	0.390	0.307	0.252	0.215	0.188	0.149	0.123	0.105
2m=7	*5%*	0.508	0.372	0.293	0.242	0.206	0.179	0.142	0.117	0.100
	1%	0.556	0.413	0.372	0.271	0.231	0.203	0.161	0.133	0.114
2m=8	*5%*	0.531	0.393	0.311	0.257	0.219	0.191	0.152	0.126	0.108
	1%	0.578	0.433	0.345	0.287	0.245	0.214	0.171	0.142	0.122
2m=9	*5%*	0.550	0.411	0.372	0.272	0.233	0.203	0.162	0.135	0.115
	1%	0.595	0.451	0.362	0.302	0.259	0.227	0.181	0.151	0.129
2m=10	*5%*	0.568	0.429	0.348	0.286	0.246	0.215	0.172	0.143	0.123
	1%	0.612	0.468	0.378	0.317	0.272	0.239	0.191	0.159	0.137
2m=12	*5%*	0.601	0.461	0.373	0.314	0.270	0.237	0.191	0.160	0.137
	1%	0.643	0.500	0.407	0.344	0.297	0.262	0.211	0.177	0.152
2m=14	*5%*	0.630	0.490	0.400	0.339	0.293	0.259	0.209	0.175	0.151
	1%	0.671	0.527	0.434	0.369	0.320	0.283	0.229	0.192	0.166
2m=16	*5%*	0.654	0.515	0.425	0.362	0.315	0.278	0.226	0.190	0.164
	1%	0.695	0.551	0.458	0.392	0.342	0.303	0.247	0.208	0.180
2m=18	*5%*	0.676	0.539	0.448	0.383	0.335	0.297	0.242	0.205	0.177
	1%	0.716	0.574	0.481	0.413	0.362	0.322	0.264	0.223	0.193
2m=20	*5%*	0.696	0.560	0.469	0.403	0.353	0.315	0.258	0.219	0.190
	1%	0.735	0.594	0.501	0.433	0.380	0.339	0.279	0.237	0.206

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